THE EFFECT OF THERMAL DIFFUSION ON THE CONDENSATION OF SMALL ADDITIONS OF VAPOR FROM A GAS FLOW

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We examine the effect of thermal diffusion on the condensation of small additions of heavier vapor molecules from the gas flow streamlining a plate. We demonstrate that the thermal diffusion in this case may noticeably increase the diffusion flow of the vapor through the laminar boundary layer to the condensation surface (the surface of the plate).

The condensation of a vapor from a vapor–gas mixture on the surface of a colder body is governed by the diffusion of the vapor through the boundary layer. The vapor diffusion flow g_1 to the condensation surface can be written in the following form (with consideration of the Stefan flow) [1, 2]:

$$g_1 = -\rho D_{12} \frac{1}{1 - \frac{p_1}{P}} \left(\frac{\partial C_1}{\partial y} + \frac{M_1}{M} \frac{k_t}{T} \frac{\partial T}{\partial y} \right). \tag{1}$$

The flow g_2 of the second noncondensed component is equal to zero. For larger relative vapor pressures $p_1/P = MC_1/M_1$ the cofactor $1/(1 - p_1/P) \approx 1$ and formula (1) is simplified.* The first term in the parentheses in (1) determines the concentration diffusion, while the second term in the parentheses determines the thermal diffusion. If the vapor molecules (component 1) are greater in mass or in size than the molecules of the noncondensed gas, the thermal-diffusion ratio K_t is positive (at temperatures exceeding the inversion temperature for thermal diffusion). This means that the vapor molecules will move as a consequence of thermal diffusion to the colder condensation surface, while the molecules of the noncondensed gas will move in the opposite direction. The total flow of the noncondensed gas molecules (resulting from concentration diffusion, thermal diffusion, and Stefan flow) will in this case be equal to zero. However, the flow of the vapor toward the condensation surface as a result of thermal diffusion increases, and the condensation rate is greater. If the vapor molecules are smaller in mass or in size than the molecules of the noncondensed gas, the relationships are the opposite. The thermal-diffusion flow of the vapor is directed from the surface, and the condensation rate is reduced.

In the following we will examine the first case – vapor molecules greater in mass (or in size) than the molecules of the noncondensed gas. The content of the vapor in the noncondensed gas will be assumed to be small. For small contents of the component 1, as follows from the relationships given in [1, 2]:

$$k_t \approx A \frac{p_1}{P}. \tag{2}$$

As shown by calculation, the quantity A for temperatures considerably in excess of the inversion temperature is a virtual constant number that is a function of the properties of the components making up the mixture. In particular, the greater the difference in the molecular weights of the components, the higher the value of A.

For small p_1/P expression (1) in conjunction with (2) is rewritten as follows:

$$g_1 = -\rho D_{12} \left(\frac{\partial C_1}{\partial y} + A \frac{C_1}{T} \frac{\partial T}{\partial y} \right). \tag{3}$$

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*Here we can neglect the effect of the Stefan flow on the flow in the boundary layer.

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Using this expression, let us evaluate the effect of thermal diffusion on the condensation of small additions of a heavier vapor from the gas flow. The concentration surface is the surface of a semibounded plate longitudinally streamlined by this flow. The boundary layer is laminar. The diffusion equation for the boundary layer is written as follows:

$$\rho u \frac{\partial C_1}{\partial x} + \rho v \frac{\partial C_1}{\partial y} = \frac{\partial}{\partial y}\left[\rho D_{12}\left(\frac{\partial C_1}{\partial y} + A\frac{C_1}{T}\frac{\partial T}{\partial y}\right)\right]. \tag{4}$$

To this equation we have to add the equation of gas motion in the boundary layer (we will assume the flow velocity to be small in comparison with the speed of sound), the equation of continuity, and the heat-transfer equation [3]. In solving this problem it is convenient to use the Dorodnitsin variables. For a longitudinally streamlined plate the Dorodnitsin variables $\xi = x$ and $\eta = \int\limits_0^y (\rho/\rho_\infty)dy$, where ρ_∞ is the constant density in the external flow. If we assume that $\rho D_{12} \sim T$ (i.e., $D_{12} \sim T^2$, since in the approximation of an ideal gas $\rho \sim 1/T$)* and if we accomplish the transformation to the Dorodnitsin variables, the diffusion equation assumes the form

$$u\frac{\partial C_1}{\partial \xi} + \tilde{v}\frac{\partial C_1}{\partial \eta} = D_{12\infty}\left[\frac{\partial^2 C_1}{\partial \eta^2} + A\frac{\partial}{\partial \eta}\left(\frac{C_1}{T}\frac{\partial T}{\partial \eta}\right)\right], \tag{5}$$

where $\tilde{v} = u(\partial\eta/\partial x) + \rho v$; $D_{12\infty}$ is the constant diffusion coefficient, referred to the temperature of the external flow.

The equations of motion, continuity, and heat transfer [3] are subject to analogous transformation. In this case, if we assume that $\mu \sim T$ and $\lambda \sim T$, the equations will include only those values of $\nu_\infty = \mu_\infty/\rho_\infty$ and $a_\infty = \lambda_\infty/\rho_\infty c_p$ that have been referred to the temperature of the external flow (the constant).† As is well known, it follows from the equations of continuity and motion that

$$u = \frac{1}{2}U_\infty\varphi'(\zeta), \quad \tilde{v} = \frac{1}{2}\sqrt{\frac{\nu_\infty U_\infty}{\xi}}\,[\zeta\varphi'(\zeta) - \varphi(\zeta)],$$

where $\zeta = (\eta/2)\sqrt{U_\infty/\nu_\infty\xi}$, while the function $\varphi(\zeta)$ satisfies the equation and the boundary conditions

$$\varphi''' + \varphi\varphi'' = 0,$$
$$\varphi = 0,\ \varphi' = 0 \text{ when } \zeta = 0,\ \varphi' \to 2 \quad \text{as} \quad \zeta \to \infty.$$

The numerical solutions obtained by Blasius and Howarth and others [3] yield values for φ, φ', and φ''.

Introducing the variables $\Phi = C_1/(C_{1\infty} - C_{1w})$ and $\theta = T/(T_\infty - T_w)$ and considering the expressions for u and $\tilde{v}$, we transform Eq. (5) to the form

$$\Phi'' + \left[\mathrm{Pr}_{D_\infty}\varphi + A\frac{\theta'}{\theta}\right]\Phi' + A\left[\frac{\theta''}{\theta} - \frac{\theta'^2}{\theta^2}\right]\Phi = 0. \tag{6}$$

The boundary conditions: $\Phi = \Phi_w = C_{1w}/(C_{1\infty} - C_{1w})$ for $\zeta = 0$, and $\Phi \to C_{1\infty}/(C_{1\infty} - C_{1w})$ as $\zeta \to \infty$.

Equation (6) is linear, with the coefficients $P(\zeta) = \mathrm{Pr}_{D\infty}\varphi + A\theta'/\theta$ and $Q(\zeta) = A[\theta''/\theta - \theta'^2/\theta^2]$, and these are functions of ζ (since θ is also a function of ζ). For the case in which $A = 0$ (without thermal diffusion) the equation assumes the form

$$\Phi_1'' + \mathrm{Pr}_{D_\infty}\varphi\Phi_1' = 0, \tag{7}$$

where $\Phi_1 = \Phi - (C_{1w})/(C_{1\infty} - C_{1w}) = (C_1 - C_{1w})/(C_{1\infty} - C_{1w})$. When $\zeta = 0$, $\Phi_1 = 0$, as $\zeta \to \infty$, $\Phi_1 \to 1$. The solution to Eq. (7) is the following [3]:

$$\Phi_1(\zeta) = \frac{\int\limits_0^\zeta [\varphi'']^{\mathrm{Pr}_{D_\infty}} d\zeta}{\int\limits_0^\infty [\varphi'']^{\mathrm{Pr}_{D_\infty}} d\zeta}. \tag{8}$$

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*As a matter of fact, D_{12} is a weaker function of T ($D_{12} \sim T^{1.5-1.8}$), so that some error is committed here.

†The functional relationships associating temperature with μ and λ are also weaker.

The solution for the heat-transfer equation is of analogous form:

$$\vartheta(\zeta) = \theta(\zeta) - \frac{T_w}{T_\infty - T_w} = \frac{T - T_w}{T_\infty - T_w} = \frac{\int_0^\zeta [\varphi'']^{\mathrm{Pr}_\infty} d\zeta}{\int_0^\infty [\varphi'']^{\mathrm{Pr}_\infty} d\zeta}. \qquad (9)$$

Equation (6) was solved numerically in conjunction with (9) on a Promin digital computer with a standard punchcard program. Gol'dberg and Mikhailov did the calculations.

The vapor pressure at the condensation surface can be assumed to be equal to the saturation pressure at the surface temperature. The calculations were performed for the case in which the saturation pressure at the surface temperature is considerably lower than the vapor pressure in the flow (and this, in turn, is small in comparison with the total pressure).* We therefore assumed that $C_{1w} = 0$. Thus, $\Phi = \Phi_1$ and when $\zeta = 0$, $\Phi = 0$ and as $\zeta \to \infty$, $\Phi \to 1$. The quantity $T_w/T_\infty - T_w$ was assumed to be equal to unity, i.e., $\theta = \vartheta + 1$. When $\zeta = 0$, $\theta = 1$, and as $\zeta \to \infty$, $\theta \to 2$. The values of the Prandtl numbers for the conditions of the external flow were assumed to be the following: $\mathrm{Pr}_{D\infty} = 2.56$ and $\mathrm{Pr}_\infty = 0.64$. This corresponds approximately to helium with small additions of cesium vapor (the viscosity, thermal conductivity, and density are determined for the helium, and the diffusion coefficient corresponds to the Cs–He binary mixture) at a temperature of $T_\infty = 800$°K. The diffusion coefficient has been determined by the data in [4]. The calculations were performed for the following values of the parameter A: A = 0 (without thermal diffusion), A = 0.98, 1.5, and 2. The value A = 0.98 corresponds to the thermal diffusion of cesium in helium. It follows from the calculation data of [5] that for small cesium contents in the case of temperatures above 400°K.

$$A = \frac{k_t}{p_1/P} \approx 0.98.$$

We used the interpolational functions in the integration of (6) for the quantities $\varphi(\zeta)$, $\varphi'(\zeta)$, $\varphi''(\zeta)$, $P(\zeta)$, and $Q(\zeta)$. For the last two quantities the greatest interpolation error is about ±3%. The functions $\varphi(\zeta)$, $\varphi'(\zeta)$, and $\varphi''(\zeta)$ are interpolated more exactly (the error does not exceed ±1-2%). In the integration we specified the values of the derivative $\Phi'(0)$ for $\zeta = 0$ until we were able to satisfy the condition $\Phi \to 1$ as $\zeta \to \infty$ (in practical terms, $\Phi = 1$ was reached when $\zeta \approx 2$). The slight deviations in the derivative $\Phi'(0)$ led to pronounced differences in the values of Φ from unity when $\zeta \approx 2$ (and for larger values of ζ). The values of the derivative $\Phi'(0)$ for the various values of A are thus determined from the error that only slightly exceeds the calculation error brought about by the interpolation of the functions. The resulting values of the derivative $\Phi'(0)$ for various values of A are the following

$$\begin{array}{lllll} A & = 0; & 0{,}98; & 1{,}5; & 2{,}0 \\ \Phi'(0) & = 0{,}92; & 1{,}1; & 1{,}2; & 1{,}3. \end{array}$$

The vapor flow to the condensation surface is proportional to the derivative $\Phi'(0)$. Consequently, under the conditions examined here, the vapor flow – on reaching the surface – increases as a consequence of thermal diffusion by a factor of 1.2 when A = 0.98, and by a factor 1.3 when A = 1.5; it increases by a factor of 1.4 when A = 2. It should be noted that near the surface the thermal diffusion, in and of itself, is weak (a low value for $k_t \approx Ap_1/P$). However, the concentration (pressure) gradient for the vapor increases because of the effect of thermal diffusion at some distance from the surface. On the whole, the calculations show that thermal diffusion can markedly increase the diffusion flow of heavy vapor molecules toward the condensation surface.

NOTATION

g_1, g_2	are the specific diffusion flows of the components (in units of mass);
ρ	is the gas density;
D_{12}	is the diffusion coefficient for the binary mixture;
P, p_1	are the total pressure and the partial pressure of the condensed component (of the vapor);
$C_1 = M_1p_1/MP$	is the relative mass concentration of the vapor;
M_1, M	are the molecular weights of the vapor and of the mixture;
x, y	are the longitudinal and transverse coordinates;

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*In addition, we assume the vapor pressure in the flow to be so small that there is no zone of extreme supersaturated vapor at the condensation surface nor does volume condensation begin (fog formation) [6].

k_t	is the thermal diffusion ratio;
u, v	are the projections of the velocity onto the x- and y-axes;
T_∞, U_∞, $T_{1\infty}$	are the values of the temperature, the velocity, and the concentration of the vapor for the free stream;
T_W, C_{1W}	are the values of the surface temperature and the vapor concentration of the surface;
ρ_∞, $D_{12\infty}$, μ_∞, ν_∞, λ_∞, a_∞	are the values for the density, the diffusion coefficient, and the coefficients of viscosity thermal conductivity, and thermal diffusivity for the free stream;
c_p	is the specific heat capacity of the gas;
$Pr_{D\infty} = \nu_\infty / D_{12\infty}$	is the Prandtl diffusion number (the Schmidt number), referred to the free-stream temperature;
$Pr_\infty = \nu_\infty / a_\infty$	is the thermal Prandtl number, referred to the free-stream temperature.

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